

High multiplicity QCD amplitudes with NJet

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NLO QCD calculations

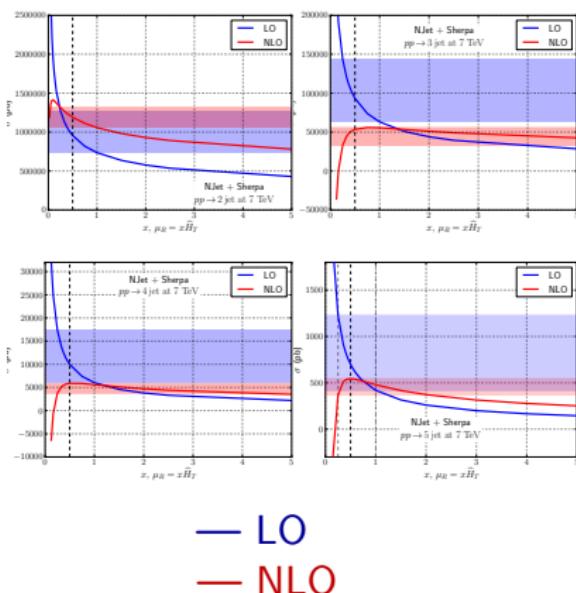
NLO results provide more accurate predictions and theoretical uncertainties for multi-jet backgrounds in new physics searches.

NLO vs LO

- ▶ Reduced theoretical uncertainty

NLO automation

- ▶ Great advances in the recent years
- ▶ High-multiplicity still remains a challenge



Hard process ingredients

$$\sigma^{\text{NLO}} = \int_n \left(d\sigma_n^B + d\sigma_n^V \right) + \int_1 \left(d\sigma_{n+1}^S \right) + \int_{n+1} \left(d\sigma_{n+1}^R - d\sigma_{n+1}^S \right)$$

↑
bottleneck

↑
bottleneck

Complicated pieces

1. **Virtual matrix elements** [NJET, QCDCLoop]
 - ▶ Integration over loop momentum
 - ▶ A number of new competing advanced methods
2. **Real + subtraction** [Sherpa, Comix]
 - ▶ Tree-like
 - ▶ Difficult phase-space integration
3. Linked with BLHA interface

NJet version 2.0¹

Multi-parton **matrix elements** in massless QCD [arXiv:1209.0100]

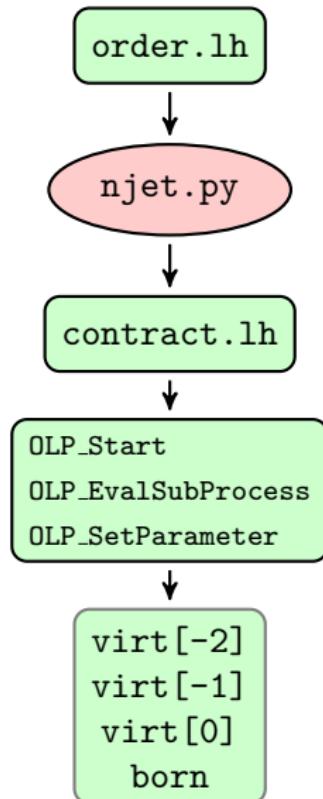
- ▶ Full colour-summed amplitudes for up to 5 outgoing partons
- ▶ Reliable accuracy estimate and rescue system
- ▶ BLHA interface for MC generators

New in version 2.0

- ▶ $W^\pm/Z/\gamma$ with up to **5 jets** and $\gamma\gamma$ with up to **4 jets**.
- ▶ Leading/Subleading colour splitting.
- ▶ Hardware vectorization for scaling test.
- ▶ BLHA2 support.
- ▶ Fast analytic amplitudes for 2 and 3 jets.

¹available from project homepage <https://bitbucket.org/njet/njet>

Binoth Les Houches Accord interface to One Loop matrix elements



BLHA

- ▶ Simple uniform interface between Monte-Carlo (MC) and One Loop Providers (OLP)

[arXiv:1001.1307, arXiv:1308.3462]

BLHA in NJet 2.0

- ▶ Support BLHA1 and BLHA2
- ▶ Control all settings via order file
- ▶ Provide colour/spin-correlated trees
- ▶ Provide leading/subleading colour and desymmetrized amplitudes

Keeping accuracy under control

Loop amplitudes

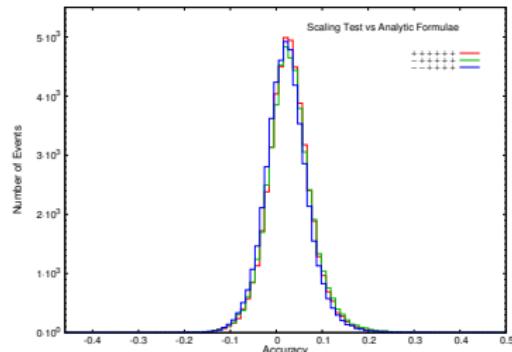
- ▶ Loop amplitudes lose accuracy in special kinematic regions.
- ▶ Tracking these regions gets harder with more legs.

NJet strategy

- ▶ Use universal scaling test to detect catastrophic cancellations.
- ▶ Re-evaluate failed points in higher precision. [libqd]

Scaling test

- ▶ Evaluate twice and compare.
- ▶ Parallelized with SSE. [libVc]
- ▶ Overall $< 10\%$ slowdown.



Advanced methods for computing amplitudes

- ▶ On-shell methods to avoid unphysical degrees of freedom
(amplitudes from trees, rational terms from massive loop cuts)
- ▶ Efficient recursive construction of building blocks
- ▶ Relations between primitive amplitudes

Time per phase-space point for dominating channels

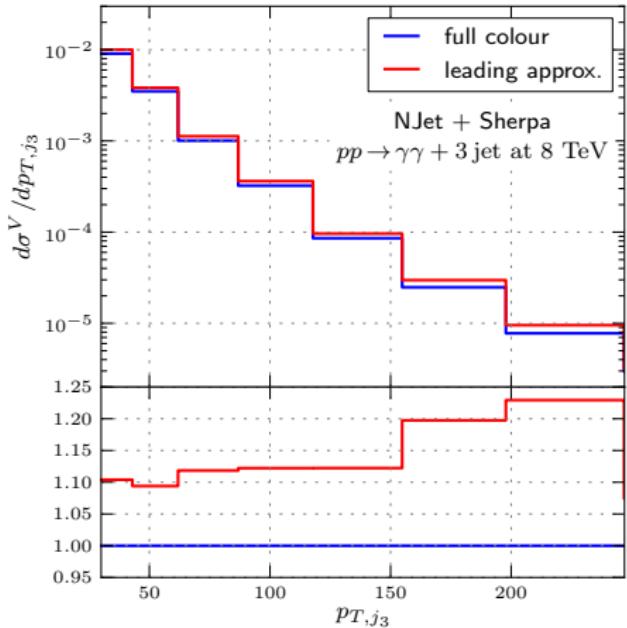
$$T(n) \sim 2^n n^6 [n!], \quad n - \text{number of final states}$$

Getting rid of the factorial (offload to MC)

- ▶ Desymmetrizing final states (available in NJet)
- ▶ Separate integration of leading/subleading colour (available in NJet)
- ▶ Colour-dressed approach (available in Sherpa/COMIX)
- ▶ Numerical loop integration

Why split into leading/subleading colour (at high multiplicity)

Subleading colour



- ▶ Order of magnitude **slower**
- ▶ Order of magnitude **smaller**
- ▶ Often cannot be ignored

Separate integration

- ▶ Full colour 5–10 times **faster**

Disadvantages

- ▶ Manual (no MC support)
- ▶ μ_R dep. has to be corrected
- ▶ Not standardized in BLHA

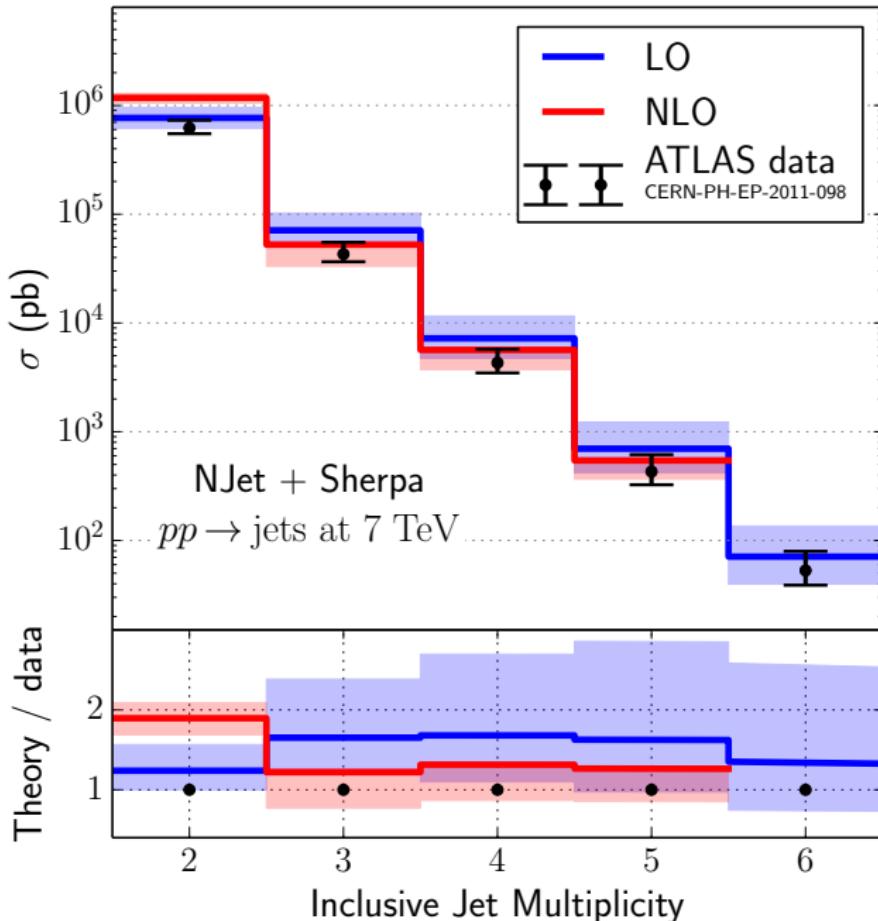
High multiplicity calculations are **expensive**

- ▶ Typical 5 final state calculations take $\sim 10^5$ CPU · hours.
- ▶ Do not want to run it more than once.

Layered computation set-up

- ▶ Save generated events in ROOT Ntuples. [arXiv:1003.1241]
- ▶ Analyze later (still several days per analysis).
- ▶ Interpolation grids with **APPLgrid** for fast PDF convolution and scale variations. [arXiv:1312.4460]

NJet+Sherpa: total XS for 2, 3, 4, 5 jets at 7 TeV vs ATLAS measurements



Cuts

anti-kt $R = 0.4$

$p_T^{1\text{st}} > 80 \text{ GeV}$

$p_T^{\text{other}} > 60 \text{ GeV}$

$|\eta| < 2.8$

NLO

$\mu_R = \mu_F = \hat{H}_T/2$

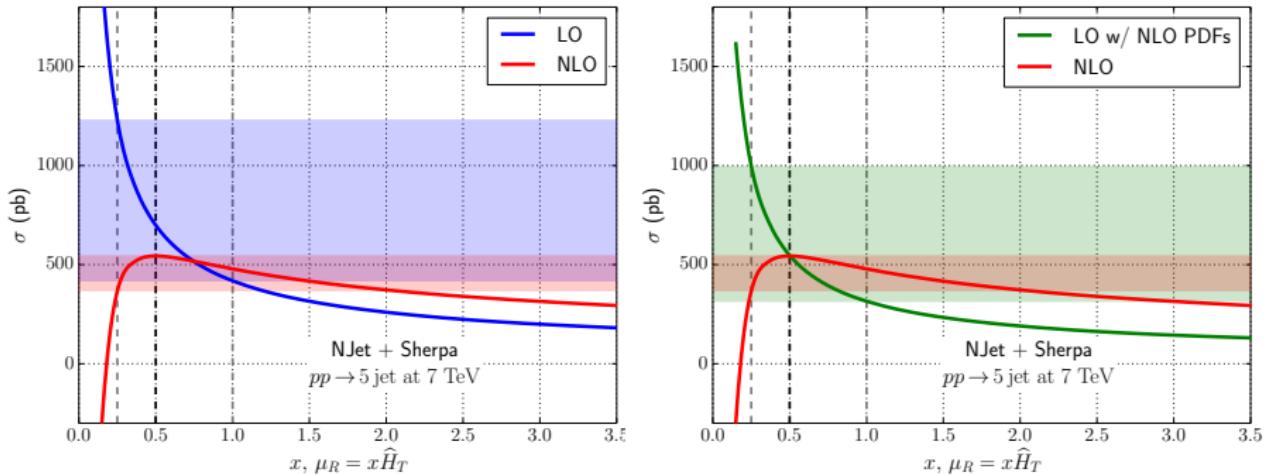
vars. $\hat{H}_T/4$ and \hat{H}_T

$\alpha_s(M_Z) = 0.118$

NNPDF23 PDF set

NJet+Sherpa: 5 jets at 7 TeV, scale variations

ATLAS cuts, NNPDF23 PDF set, $\alpha_s(M_Z) = 0.118$



$$\sigma_5^{\text{7TeV-LO}}(\mu = \hat{H}_T/2) = 0.699(0.004)^{+0.530}_{-0.280} \text{ nb}$$

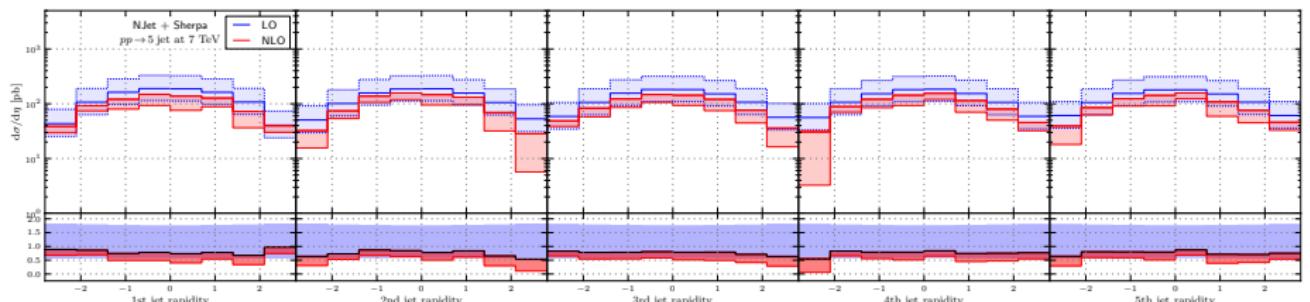
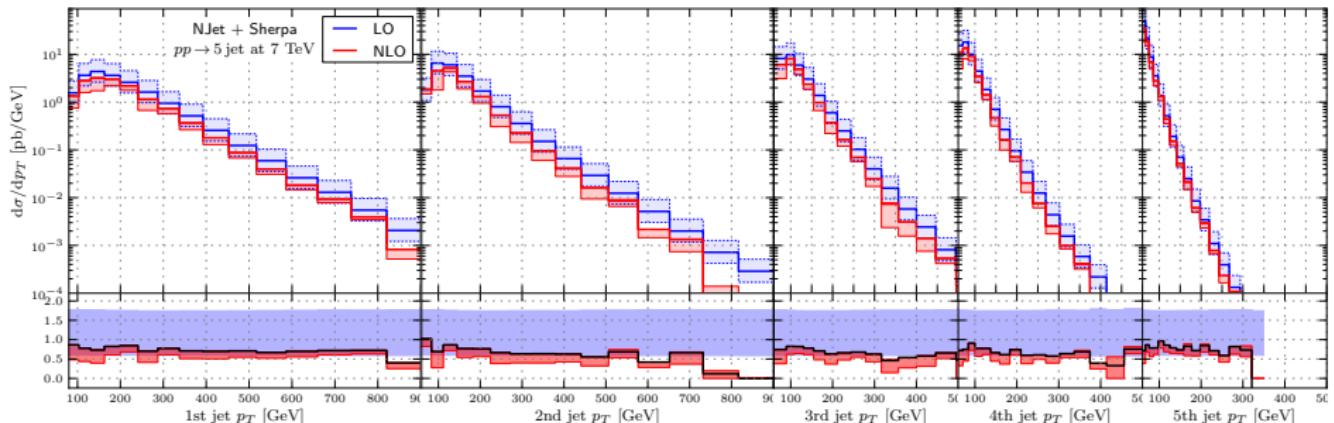
$$\sigma_5^{\text{7TeV-NLO}}(\mu = \hat{H}_T/2) = 0.544(0.016)^{+0.0}_{-0.177} \text{ nb}$$

$$\sigma_5^{\text{8TeV-LO}}(\mu = \hat{H}_T/2) = 1.044(0.006)^{+0.770}_{-0.413} \text{ nb}$$

$$\sigma_5^{\text{8TeV-NLO}}(\mu = \hat{H}_T/2) = 0.790(0.021)^{+0.0}_{-0.313} \text{ nb}$$

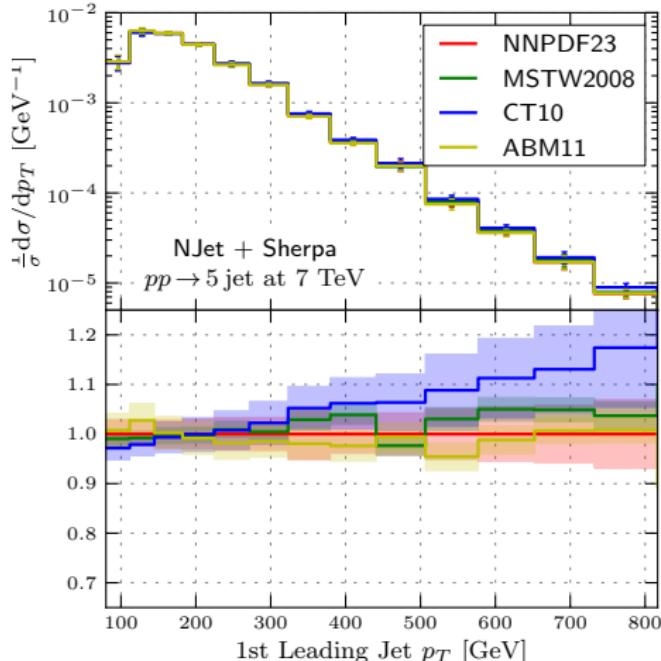
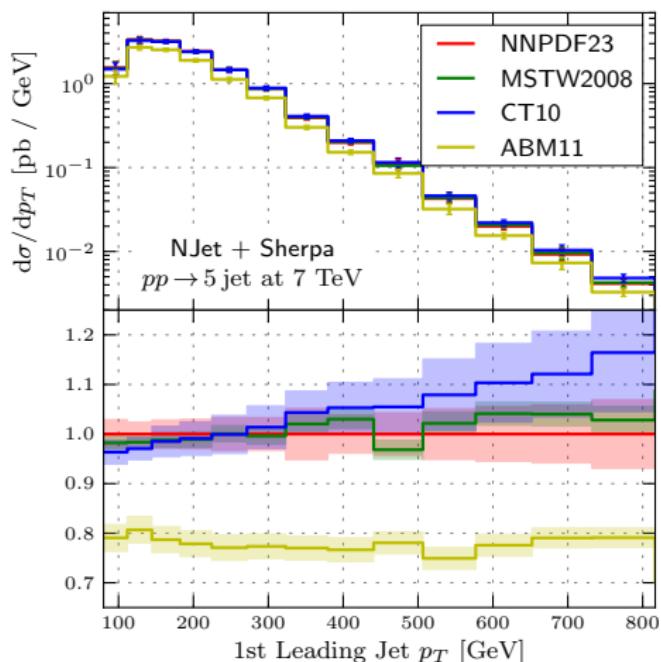
NJet+Sherpa: 5 jets at 7 TeV, p_T and η distributions

ATLAS cuts, NNPDF23 PDF set, $\alpha_s(M_Z) = 0.118$



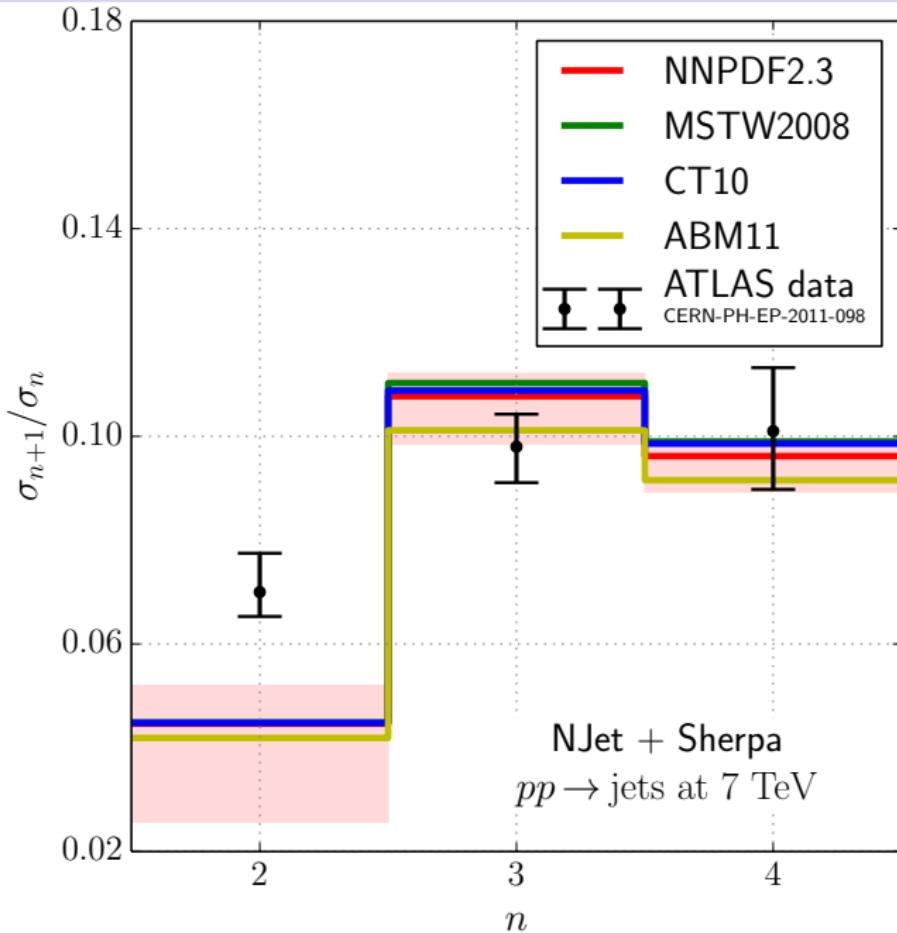
NJet+Sherpa: 5 jets at 7 TeV, PDF uncertainties

ATLAS cuts, $\alpha_s(M_Z) = 0.118$, PDF uncertainty $\approx 3\%$



Right plot — distributions normalized to total cross-section.

NJet+Sherpa: jets ratios at 7 TeV with different PDFs vs ATLAS data



Cuts

anti-kt $R = 0.4$

$p_T^{\text{1st}} > 80 \text{ GeV}$

$p_T^{\text{other}} > 60 \text{ GeV}$

$|\eta| < 2.8$

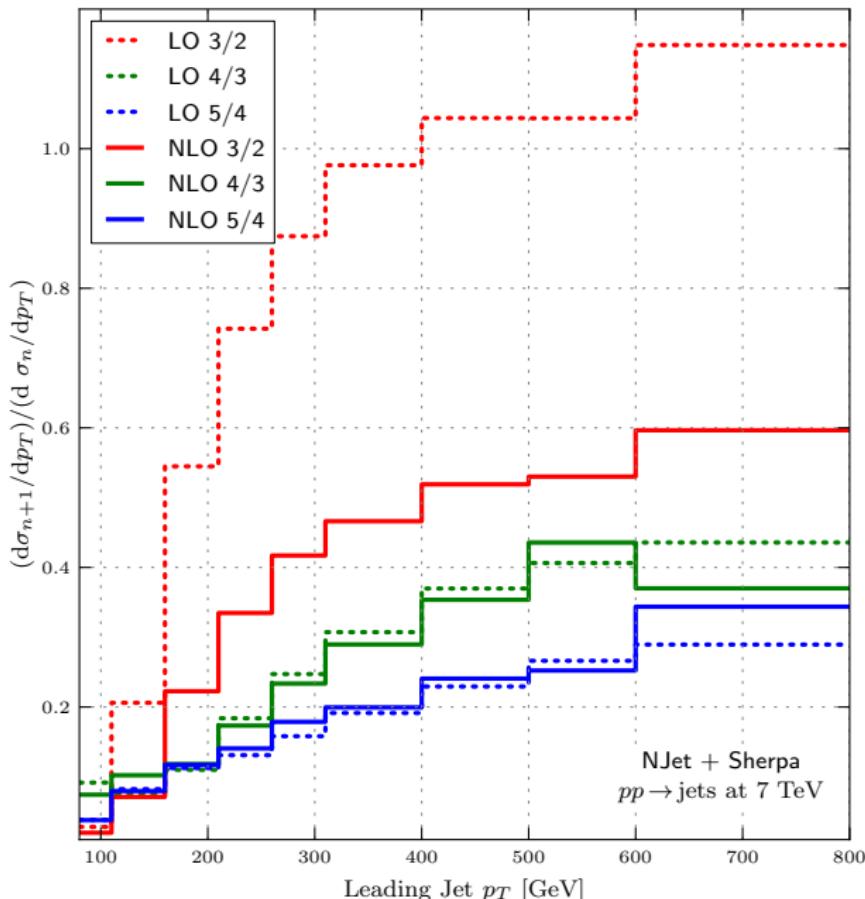
NLO

$\mu_R = \mu_F = \hat{H}_T/2$

vars. $\hat{H}_T/4$ and \hat{H}_T
(shown for NNPDF)

$\alpha_s(M_Z) = 0.118$

NJet+Sherpa: p_T for jets ratios at 7 TeV



Cuts

anti-kt $R = 0.4$

$p_T^{1\text{st}} > 80 \text{ GeV}$

$p_T^{\text{other}} > 60 \text{ GeV}$

$|\eta| < 2.8$

NLO

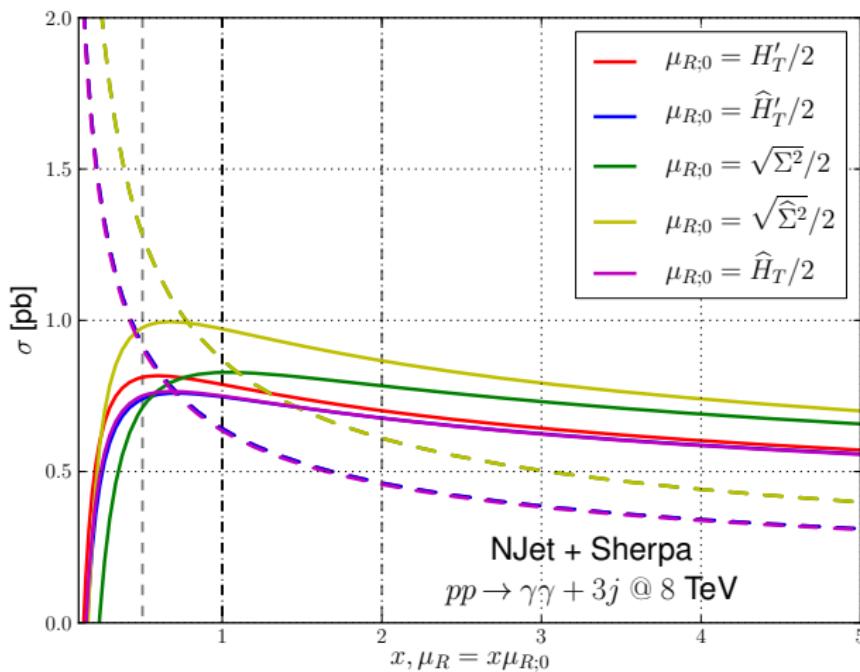
$\mu_R = \mu_F = \hat{H}_T/2$

vars. 60%, 12%, 9%
 (not shown)

$\alpha_s(M_Z) = 0.118$

NNPDF23 PDF set

NJet+Sherpa: $\gamma\gamma + 3j$ at 8 TeV, scale variations, CT10nlo PDF



$$\sigma_{\gamma\gamma+3j}^{LO}(\hat{H}'_T/2) = 0.643(0.003)^{+0.278}_{-0.180} \text{ pb}$$

$$\sigma_{\gamma\gamma+3j}^{NLO}(\hat{H}'_T/2) = 0.785(0.010)^{+0.027}_{-0.085} \text{ pb}$$

Cuts

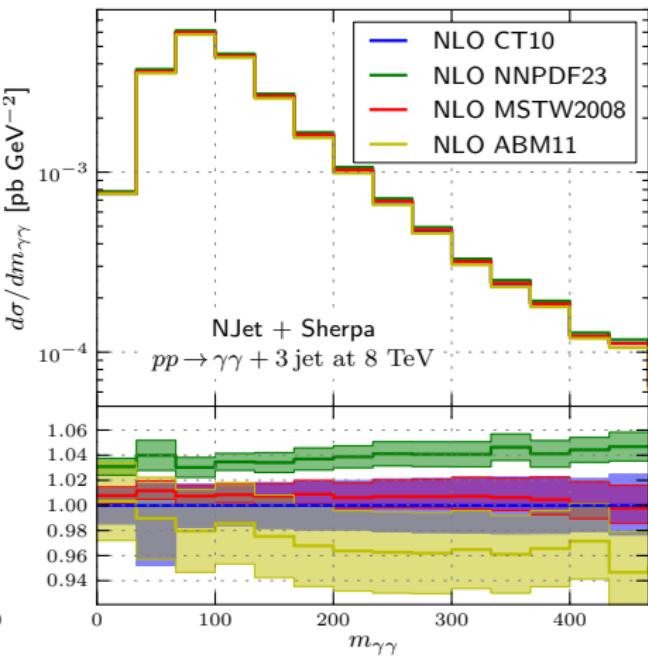
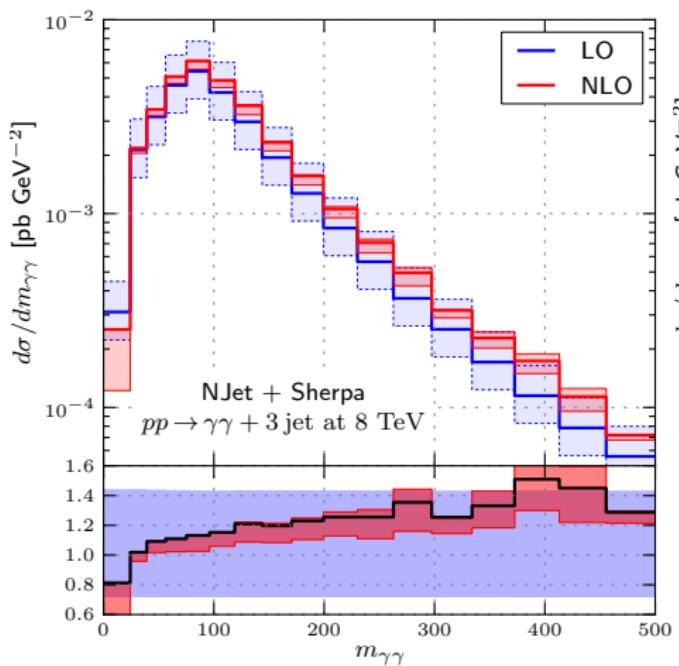
- $p_{T,j} > 30 \text{ GeV}$
- $|\eta_j| \leq 4.7$
- $p_{T,\gamma_1} > 40 \text{ GeV}$
- $p_{T,\gamma_2} > 25 \text{ GeV}$
- $|\eta_\gamma| \leq 2.5$
- $R_{\gamma,j} = 0.5$
- $R_{\gamma,\gamma} = 0.45$

Scales

- $\hat{H}_T = \sum_{i \in \{\gamma, \text{partons}\}} p_{T,i}$
- $\hat{H}'_T = m_{\gamma\gamma} + \sum_{i \in \text{partons}} p_{T,i}$
- $\hat{\Sigma}^2 = m_{\gamma\gamma}^2 + \sum_{i \in \text{partons}} p_{T,i}^2$
- $H'_T = m_{\gamma\gamma} + \sum_{i \in \text{jets}} p_{T,i}$
- $\Sigma^2 = m_{\gamma\gamma}^2 + \sum_{i \in \text{jets}} p_{T,i}^2$

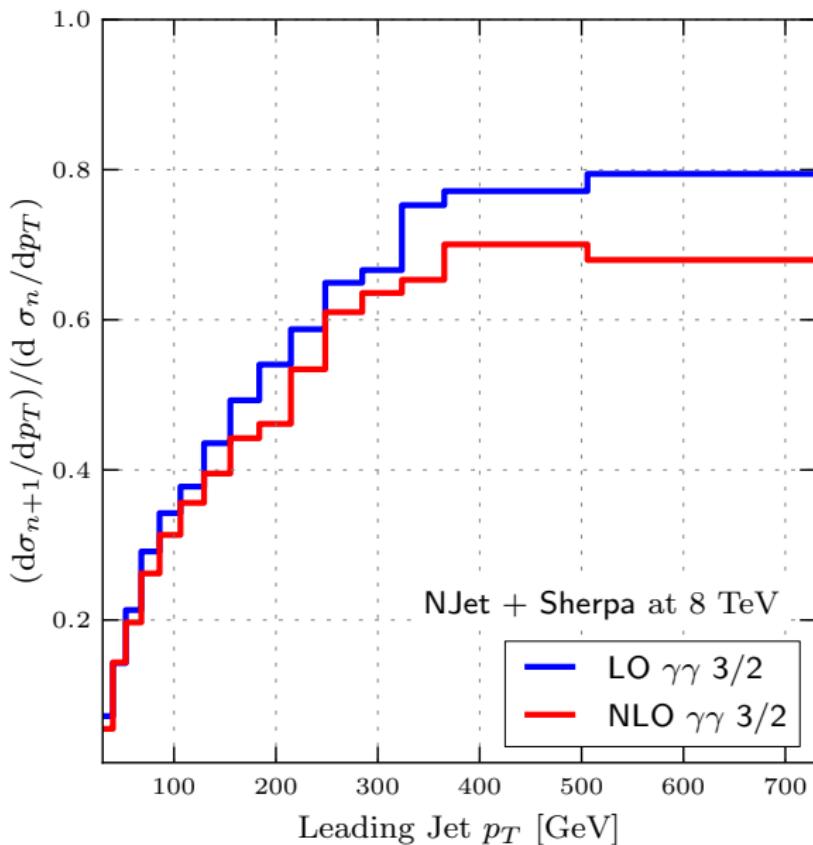
NJet+Sherpa: $\gamma\gamma + 3j$ at 8 TeV, $m_{\gamma\gamma}$ distribution and PDF uncertainties

PDF uncertainty $\approx 3\text{--}6\%$



Di-photon invariant mass distribution

NJet+Sherpa: p_T for $\gamma\gamma + \text{jets}$ 3/2 ratio at 8 TeV



Ratio

$$\mu_R = \mu_F = \hat{H}'_T/2$$

$$R_{3/2}^{LO} = 0.314(0.002)$$

$$R_{3/2}^{NLO} = 0.276(0.004)$$

Scale

Different scales
agree within
8% for $R_{3/2}$

Conclusions

Summary

- ▶ NJet library version 2 with improved speed and new processes
- ▶ NJet+Sherpa: $pp \rightarrow 3, 4, 5$ jets at NLO at 7 and 8 TeV
- ▶ NJet+Sherpa: $pp \rightarrow \gamma\gamma + 2, 3$ jets at NLO at 8 TeV

Outlook

- ▶ Today: fully automated 4 final state predictions at NLO
- ▶ In a few years: routine calculations with 5/6 final states
- ▶ Going beyond would require new methods/ideas

Bonus material

Desymmetrized amplitudes

Observation

- ▶ Squared amplitudes are **totally symmetric** over final state gluons
- ▶ Gluon phase space integration is a **symmetric operator**

Idea

- ▶ Replace squared amplitudes with something simpler
(specialized **full colour** sum, no change on the MC side)

Example:

$$\iiint_a^b (x^2y + x^2z + xy^2 + xz^2 + y^2z + yz^2) dx dy dz = \iiint_a^b 6x^2y dx dy dz$$

Get the same result $n_g!/2$ times cheaper

	$gg \rightarrow 3g$	$gg \rightarrow 4g$	$gg \rightarrow 5g$
Standard sum	0.22 s	6.19 s	171.31 s
De-symmetrized	0.07 s	0.50 s	2.76 s
Speedup	$\times 3$	$\times 12$	$\times 60$

ROOT NTuples output

[arXiv:1003.1241]

Store in NTuples:

- ▶ Can change scales and/or PDFs during analysis
- ▶ Easy to create **APPLgrid's**
- ▶ Takes a lot of disk space
- ▶ Needs custom software for full flexibility

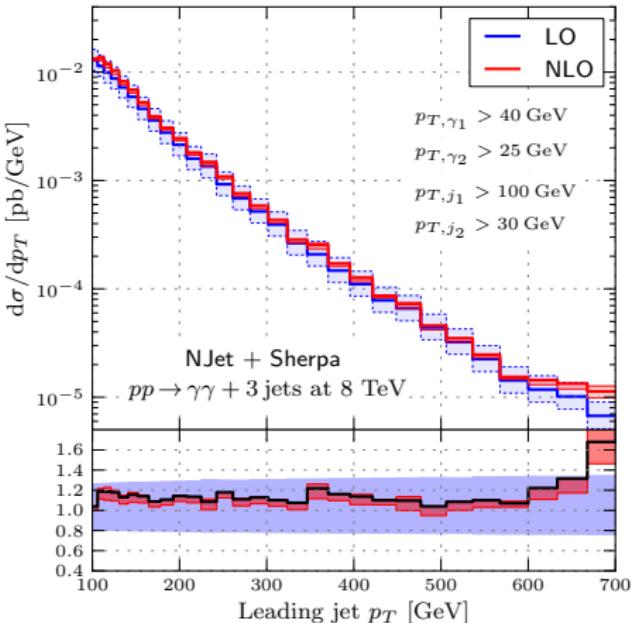
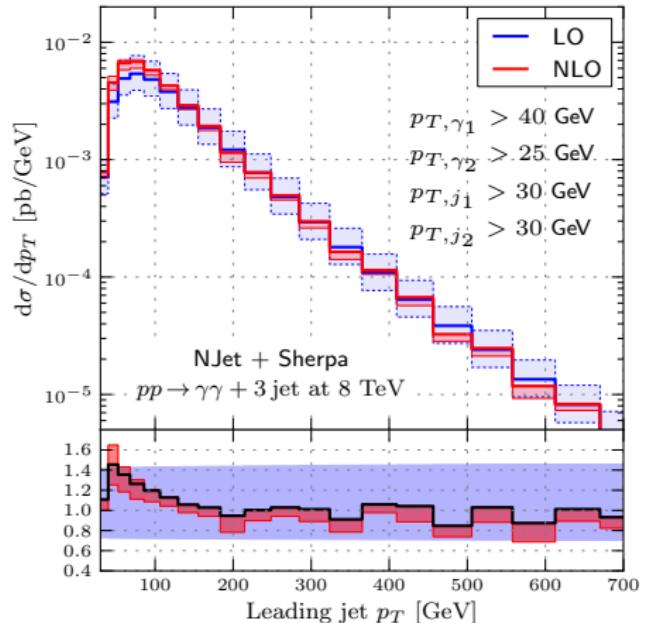
Analyze on-the-fly:

- ▶ Easy to set-up
- ▶ No need to save events
- ▶ Can use standard tools: **Rivet**
- ▶ Scale changes and PDF variations are **very expensive**

Possible improvements

- ▶ Use several Rivet analyses with different μ_R and μ_F in a single run would allow to do simple NLO calculations on-the-fly.
- ▶ Interface for custom analysis codes providing information similar to what is passed to NTuples (extend Rivet interface?)

NJet+Sherpa: $\gamma\gamma + 3j$ at 8 TeV, leading p_T cut dependence



p_{T,j_1} cut dependence in leading jet p_T distribution.

Partial Amplitudes and Colour Summation

Colour decomposition of an L-loop amplitude:

$$\mathcal{A}_n^{(L)}(\{p_i\}) = \sum_c \underbrace{T_c(\{a_i\})}_{\text{colour basis}} \underbrace{A_{n;c}^{(L)}(p_1, \dots, p_n)}_{\text{partial amplitudes}}$$

Partial amplitudes → squared matrix elements

$$|\mathcal{M}_n|^2 = \sum_{\text{hel}} \sum_{\text{col}} \mathcal{A}_n^{(L)} \mathcal{A}_n^{(0)\dagger} = \sum_{\text{hel}} \sum_{cc'} A_{n;c}^{(L)} \cdot \mathcal{C}_{cc'} \cdot A_{n;c'}^{(0)\dagger}$$

Colour matrix

$$\mathcal{C}_{cc'} = \sum_{\{a_i\}} T_c(\{a_i\}) T_{c'}(\{a_i\})$$

$$T_c(\{a_i\}) = T_{jk}^{a_1} \dots \delta_{lm} \dots$$

Partial Amplitudes and Colour Summation

Colour decomposition of a 1-loop amplitude:

Partial amplitudes are linear combinations of primitive amplitudes.

$$A_{n;c}^{(1)} = \sum_k a_{k;c} A_n^{[m]} + N_f b_{k;c} A_n^{[f]}$$

Partial-Primitive decomposition for gluons and $q\bar{q} + \text{gluons}$:

- ▶ Tree level: Kleiss-Kuijf basis of $(n - 2)!$ primitives
- ▶ One-loop: a basis of $(n - 1)!$ primitives.

[Kleiss,Kuijf], [Bern,Dixon,Dunbar,Kosower]

Partial-Primitive decomposition for multi-quark case:

No analytic formula. Reconstruct partials using diagram matching.

[Ellis,Kunszt,Melnikov,Zanderighi], [Ita,Ozeren], [NJet]

Generic Partial-Primitive decomposition

Outline of the algorithm

1. Generate all diagrams' topologies for the amplitude \mathcal{A}_n
2. Write primitives P_i as combinations of colour-stripped diagrams K_i using matching matrix M_{ij}
3. Invert the system to get partial amplitudes in terms of independent set of primitives \hat{P}

$$\mathcal{A}_n = \sum_c T_c(\{a_i\}) \sum_{j=1}^{\hat{N}_{\text{pri}}} Q_{cj} \hat{P}_j$$

Ensure linearly independent set by capturing all relations between color-ordered diagrams.